FAR BEYOND

MAT122

Product Rule

Stony Brook University

Review

Power Rule

$$(ax^n)' = \frac{d}{dx} ax^n = nax^{n-1}$$

Exponential Derivative:
(base *e*)

$$\left(e^x\right)' = e^x$$

Special Cases:

$$\frac{d}{dx} x = 1$$

$$\frac{d}{dx} a = 0$$
where a is a constant

Exponent Law:

$$\sqrt[n]{x} = x^{1/n}$$

Do: differentiate $f(x) = 5x^{100} - e^x + 7\sqrt[3]{x} + 11$

Product Rule - Intro

When two *differentiable* functions are multiplied, use the **Product Rule** to take derivative:

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

$$[f(x)g(x)]' = f'(x)\,g(x) + f(x)\,g'(x)$$

$$\frac{d}{dx}[f(x)g(x)] = \frac{d}{dx}f(x) \cdot g(x) + f(x) \cdot \frac{d}{dx}g(x)$$

ex. find the derivative of $h(x) = xe^x$

$= e^x + x\,e^x$

Product Rule – cont'd

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

ex. find the derivative of $h(x) = (x^2 + 3x)\,(5x^3 - 2)$

$$h'(x) = (2x+3)(5x^3-2) + (x^2+3x)(15x^2)$$

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ex. differentiate $f(y) = \left(\frac{1}{y^2} - 3y^4\right)\left(y + 5y^3\right)$

$$f'(y) = \left(-\frac{2}{y^3} - 12y^3\right)\left(y + 5y^3\right) + \left(\frac{1}{y^2} - 3y^4\right)\left(1 + 15y^2\right)$$

Product Rule – Do

$$(f \cdot g)' = f' \cdot g + f \cdot g'$$

Do: differentiate $f(x) = \left(4x^3 - 6x^2 + 1\right)\left(5x^4 + 7x^2 + 3x\right)$

Do: find $f'(x)$: $f(x) = \left(x + \sqrt{x} + \sqrt[3]{x}\right)\left(e^x - x^2\right)$

Rate of Change - Application

ex. The cost (in dollars) of producing x phone chargers is given by $C(x) = (3x - 25)(500 - x)$. Find the rate at which cost is changing when 100 chargers have been produced.

Step 1: find general derivative

Step 2: plug 100 into derivative